Past examination questions for topic 3

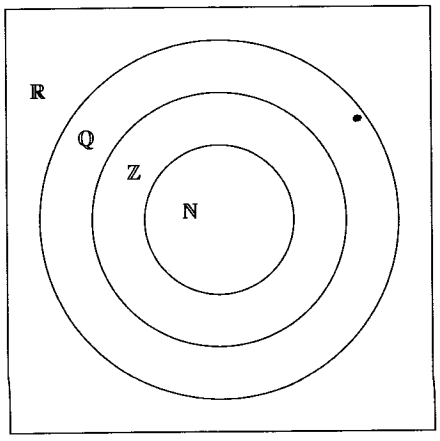
Paper 1

1 The Venn diagram shows the universal set of real numbers $\mathbb{R}$ and some of its important subsets:

$\mathbb{Q}$: the rational numbers
$\mathbb{Z}$: the integers
$\mathbb{N}$: the natural numbers.

Write the following numbers in the correct position in the diagram.

$-1,\ 1,\ \pi,\ \frac{7}{16},\ 3.333\dot{3},\ \sqrt{3}$.



M0

2 Given the statements

p: The sun is shining
q: I am wearing my hat

a Write down, in words, the meaning of $q \Rightarrow \neg p$.

b Complete the truth table

p	q	$\neg q$	$q \Rightarrow \neg p$
T	T		
T	F		
F	T		
F	F		

c Write in symbols, the converse of $q \Rightarrow \neg p$.

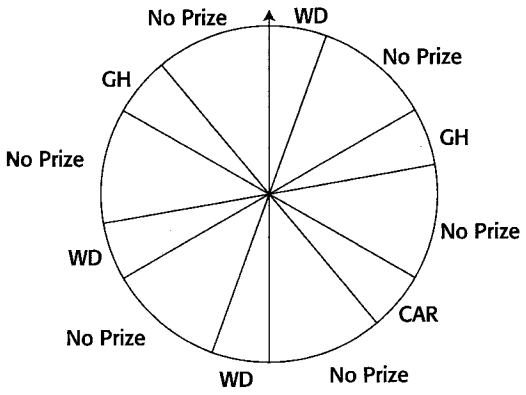
M0

3 In a group of fifteen students, three names begin with the letter B and four begin with a G. The remaining eight names begin with A, C ,D, E, F ,H, I and J respectively.
The fifteen names are placed in a box. The box is shaken and two names are drawn out.

Find the probability that

a both names begin with any letter except G or B
b both names begin with the same letter
c both names begin with the letter H. *M06q8*

4 On a certain game show, contestants spin a wheel to win a prize, as shown in the diagram. The larger angles are 40° and the smaller angles are 20°.



Find the probability that a contestant
a will not win a prize
b will win a holiday in Greece (*GH*)
c will win a washer/dryer (*WD*), given that he knows that he has won a prize
d will win a holiday to Greece *or* a washer/dryer. *M00q10*

5 Events A and B have probabilities $P(A) = 0.4$, $P(B) = 0.65$ and $P(A \cup B) = 0.85$.

a Calculate $P(A \cap B)$.
b State with a reason whether events A and B are independent.
c State with a reason whether events A and B are mutually exclusive. *Specimen Paperq13*

6 Two propositions p and q are defined as follows:
p: the number ends in zero
q: the number is divisible by 5.

a Write in words
i $p \Rightarrow q$
ii the converse of $(p \Rightarrow q)$.
b Write in symbolic form
i the inverse of $(p \Rightarrow q)$
ii the contrapositive of $(p \Rightarrow q)$. *M01q12*

Paper 2

1 Let U be the set of all positive integers from 1 to 21 inclusive.

A, B and C are subsets of U such that:

A contains all the positive integers that are factors of 21
B is the set of multiples of 7 contained in U
C is the set of odd numbers contained in U.

a List all the members of set A.

b Write down all the members of

i $A \cup B$

ii $C' \cap B$.

Find the probability that a member chosen at random from A is also a member of $A \cap B \cap C$. *M06q2(i)*

2 A school jazz band contains three different musical instruments – saxophone (S), clarinet (C) and drums (D). Students in the band are able to play one, two or three different instruments. In a class of 40 IB students, 25 belong to the jazz band. Out of these 25

3 can play all three instruments
5 can play the saxophone and clarinet *only*
5 can play *at least* the clarinet and the drums
7 can play *at least* the saxophone and drums
16 can play the saxophone
12 can play the clarinet.

a Draw a Venn diagram and clearly indicate the number of students in each region.

b Show that the number of students who play drums *only* is 5.

c Find the probability that a student chosen at random from the IB class plays only the saxophone.

d Fine the probability that a student chosen at random from the IB class plays either the clarinet or drums or both.

e Given that a student plays the saxophone, find the probability that he also plays the clarinet. *N04q1*

3 On a particular day 100 children are asked to make a note of what they drank that day. They are given three choices, water (W), coffee (C) or fruit juice (F).

1 child drank only water.
6 children drank only coffee.
8 children drank only fruit juice.
5 children drank all three.
7 children drank water and coffee only.
53 children drank coffee and fruit juice only.
18 children drank water and fruit juice only.

a Represent this information on a Venn diagram.
b How many children drank none of the drinks listed?
c A child is chosen at random. Find the probability that the child drank
   i coffee
   ii water or fruit juice but not coffee
   iii no fruit juice, given that the child did drink water.
d Two children are chosen at random. Find the probability that both children drank all three drinks. *N03q2*

4 Let F be the set of all families that have exactly two children. Assuming P(boy) = P(girl) = 0.5, find the probability that a family chosen at random from F has exactly
a two boys
b two boys, if it is known that the first child is a boy
c two boys, if it is known that there is a boy in the family. *N01q1(ii)*

4 **a** 0.36 **b** 0.6268 (0.627 3s.f.)
c 0.1788 (0.179 3s.f.)

Exercise 3.10

1 0.85

2 0.32

3 **a** P(A) = 18/60, P(B) = 18/60, P(C) = 26/60
b A and B since the intersection is zero. C and B since the intersection is zero.

4 0.0522

5 **a** 0.5 **b** 0.15 **c** 0.3 **d** 0.4
e neither **f** mutually exclusive.

6 **a** 3/11 **b** 5/11 **c** 3/5

7 **a** 0.6084 **b** 0.6854

8 **a** 0.396 **b** 0.6525

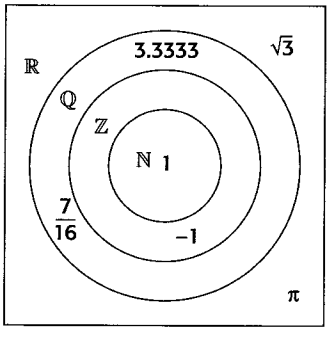
9 **a** 13/80 **b** 38/80 **c** $\frac{12}{38}$

10 **a** P(A) = 1/13, P(B) = 0.5, P(C) = 0.25, P(D) = 0.25
b 1/13 **c** 0.5
d A and D, A and C as $P(A) \times P(D) = P(A \cap D)$, $P(A) \times P(C) = P(A \cap C)$ and $P(A) \times P(B) = P(A \cap B)$
e C and D, B and D as $P(C \cap D) = 0$ and $P(B \cap D) = 0$

Answers to past examination questions topic 3

Paper 1

1



2 **a** If I am wearing my hat then the sun is not shining.

b

p	q	$\neg q$	$q \Rightarrow \neg p$
T	T	F	F
T	F	F	T
F	F	T	T
F	F	T	T

c $\neg p \Rightarrow q$

3 **a** $\frac{4}{15}$ (= 0.267 3s.f.)
b $\frac{3}{35}$ (= 0.0857 3s.f.) **c** 0

4 **a** 2/3 **b** 1/9 **c** 0.5 **d** 5/18

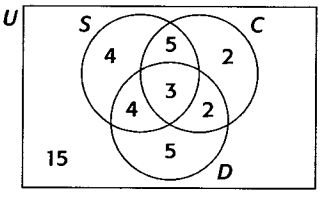
5 **a** 0.2 **b** $0.4 \times 0.65 \neq 0.2 \Rightarrow$ not independent
c $P(A \cap B) \neq 0 \Rightarrow$ not mutually exclusive

6 **a** **i** *If the number ends in zero then it is divisible by 5*
ii *If the number is divisible by 5 then it ends in zero,* (converse)
b **i** *If the number does not end in zero then it is not divisible by 5,* (inverse)
ii *If the number is not divisible by 5 then it does not end in zero,* (contrapositive)

Paper 2

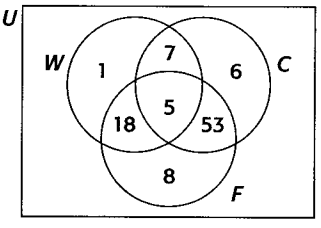
1 **a** $A = \{1, 3, 7, 21\}$
b **i** $A \cup B = \{1, 3, 7, 14, 21\}$
ii C' is the set of all even numbers in U
$C' \cap B = \{14\}$
c $A \cap B \cap C = \{7, 21\}$
P(member of A and $A \cap B \cap C$) = $\frac{1}{2}$ (0.5)

2 **a**



b 25 − (5 + 3 + 4 + 4 + 2 + 2) = 5 **c** $\frac{1}{10}$ (= 0.1)
d $\frac{21}{40}$ (= 0.525), **e** $\frac{1}{2}$ (= 0.5)

3 **a**



b 2 **c** **i** 0.71 **ii** 0.27 **iii** $\frac{8}{31}$ = 0.258 (3s.f.)
d $\frac{1}{495}$ = 0.002 02 (3s.f.)

4 **a** 0.25 **b** 0.5 **c** 1/3

Topic 4

Exercise 4.1

1 **a** Domain {−2,−1, 0, 1, 2, 3} Codomain {25, 50, 75} Range {25, 50, 75}
b Domain {10, 20, 30, 40} Codomain {50, 90, 130, 170} Range {50, 90, 130, 170}
c Domain {1, 2, 3} Codomain {5, 7, 9, 11, 13} Range {5, 9, 13}, **a** and **b** are functions.